

Meeting 4: What Is Math?

CS198: The Poetry of Computer Science, the Computer Science of Poetry
Philosophy of Computation at Berkeley
pocab.org

October 17, 2017

1 View Overview

- **Platonism:** Math is eternal, unchanging, and independent of our wet, messy world.
- **Empiricism:** Math is empirical knowledge. We know $1 + 1 = 2$ only because we deduce, from experience, that if you take an apple and take another, we end up with two apples.
- **Monism:** Max Tegmark’s view in *Our Mathematical Universe*. Goes even further than Platonism by saying math is the *only* thing that exists.
“All structures that exist mathematically also exist physically. That is, in the sense that “in those [worlds] complex enough to contain self-aware substructures [they] will subjectively perceive themselves as existing in a physically ‘real’ world””. (Wikipedia)
- **Logicism:** Math is reducible to logic, that is, math is a subset of logic. Frege started this theory. Russell and Whitehead advanced it further in *Principia Mathematica*.
- **Formalism:** Math is a “game” of string manipulation. The strings (ex: proofs or theorems) themselves are meaningless, though one may attach interpretations to them. Math is about the study of formal systems, the study of what can be deduced from formal systems: given some axioms and some rules of manipulation, what follows? Hilbert was one of the first influential formalists. Most closely related to theoretical computer science.
- **Psychological Constructivism:** Psychological constructivism says that “Learning is constructed from each individual’s experiences and connections between previously learned concepts and new ideas” (Reedal, “Jean Piaget’s Cognitive Development Theory in Mathematics Education”). The developmental psychologist Piaget’s view. Math concepts develop in the child as the child develops. As the child grasps more sophisticated concepts in the real world, so too he can grasp more sophisticated mathematical concepts. Examples: conservation of quantity, one-to-one correspondence, finding the nipple to suck on it.
- **The Theory of Embodied Math:** More radical than Piaget; math is embodied, math is derived from the body; for example, infinity is just a metaphor for some physical action done over and over again, such as walking. Lakoff and Núñez argue for this in *Where Mathematics Comes From*.

A thematic question emerges: is math independent of, or dependent on, the human mind? So to speak, does math exist “out there”, or “in here”?¹ A useful exercise: for each of the above positions, classify the position into “realism” or “anti-realism”, or justify why the position cannot be classified.

In the rest of the meeting, we will pit together two representatives, each from one of the two poles: Platonism and the Theory of Embodied Math.

¹These are called, respectively, “realism” and “anti-realism”, which, I think, are horrible names, because they presuppose an ontological commitment to what is and isn’t “real”. The presupposition is that math is “real” only insofar as it exists independently of the human mind, which is to say that whatever depends on the human mind is not “real”, which, I think, is a lot to ask for.

2 Mathematical Platonism

Here are some definitions of Mathematical Platonism:

Platonism is the doctrine that mathematical theories relate to systems of abstract objects, existing independently of us, and that the statements of those theories are determinately true or false independently of our knowledge.

(Dummett)

A mathematical realist, or platonist, (as I will use these terms) is a person who (a) believes in the existence of mathematical entities (numbers, functions, sets and so forth), and (b) believes them to be mind-independent and language-independent.

(Field)

[Platonism is] the view that mathematics describes a non-sensual reality, which exists independently both of the acts and [of] the dispositions of the human mind and is only perceived, and probably perceived very incompletely, by the human mind.

(Gödel)

And here are some comments made on it:

Mathematical platonism enjoys widespread support and is frequently considered the default metaphysical position with respect to mathematics. This is unsurprising given its extremely natural interpretation of mathematical practice. In particular, mathematical platonism takes at face-value such well known truths as that “there exist” an infinite number of prime numbers, and it provides straightforward explanations of mathematical objectivity and of the differences between mathematical and spatio-temporal entities. Thus arguments for mathematical platonism typically assert that in order for mathematical theories to be true their logical structure must refer to some mathematical entities, that many mathematical theories are indeed objectively true, and that mathematical entities are not constituents of the spatio-temporal realm.

(Internet Encyclopedia of Philosophy)

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.

(The Unreasonable Effectiveness of Mathematics on the Physical Sciences)

- Consider the following argument:

Mathematical Platonism establishes a “gap” between math as a Platonic object in itself and math as knowledge humans can only aspire to know but never fully understand. This idea is not dissimilar to the idea that there is a God and humans can only aspire to know about Him, but can never understand Him fully. Therefore, if you reject faith in God, by the same reasoning, you must reject faith in Mathematical Platonism.

Discuss the argument with your group. Lay out the assumptions and the analogy made in the argument. Is the argument sound? If not, why? If so, what follows?

3 The Theory of Embodied Math

Lakoff, a cognitive linguist and philosopher, argues that math is explained through metaphors from the body, and, for example, something as mundane as my ability to move my hand from here to here forms the basis of some of the most profound mathematical theorems. This sounds less ridiculous if one considers the rest of Lakoff's philosophy. Lakoff made a big splash in 1980 with *Metaphors We Live By*, in which he begins his influential Conceptual Metaphor Theory, by which *everything* is metaphor:

Metaphor is for most people a device of the poetic imagination and the rhetorical flourish—a matter of extraordinary rather than ordinary language. Moreover, metaphor is typically viewed as characteristic of language alone, a matter of words rather than thought or action. For this reason, most people think they can get along perfectly well without metaphor. We have found, on the contrary, that metaphor is pervasive in everyday life, not just in language but in thought and action. Our ordinary conceptual system, in terms of which we both think and act, is fundamentally metaphorical in nature.

Primarily on the basis of linguistic evidence, we have found that most of our ordinary conceptual system is metaphorical in nature. And we have found a way to begin to identify in detail just what the metaphors are that structure how we perceive, how we think, and what we do.

To give some idea of what it could mean for a concept to be metaphorical and for such a concept to structure an everyday activity, let us start with the concept ARGUMENT and the conceptual metaphor ARGUMENT IS WAR. This metaphor is reflected in our everyday language by a wide variety of expressions:

ARGUMENT IS WAR

Your claims are *indefensible*.

He *attacked every weak point* in my argument. His criticisms were *right on target*.

I *demolished* his argument.

I've never *won* an argument with him.

You disagree? Okay, *shoot!*

If you use that *strategy*, he'll *wipe you out*. He *shot down* all of my arguments.

(Lakoff & Johnson, *Metaphors We Live By*)

So it is not surprising that Lakoff would think of math as metaphor. Lakoff explicitly rejects Mathematical Platonism when he says, “the only mathematics we can know is a brain-and-mind-based mathematics.” Lakoff further argues that, while human mathematics is an object of empirical investigation, whatever mathematical Platonists believe to be “true” math is by definition outside of our investigative faculties and is thus not an empirical question, only a matter of faith.